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PWS-0182-0001

# Calibration of a Mathematical Model of the Antelope Valley Ground-Water Basin, California

By TIMOTHY J. DURBIN

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*Prepared in cooperation with the  
California Department of Water Resources*



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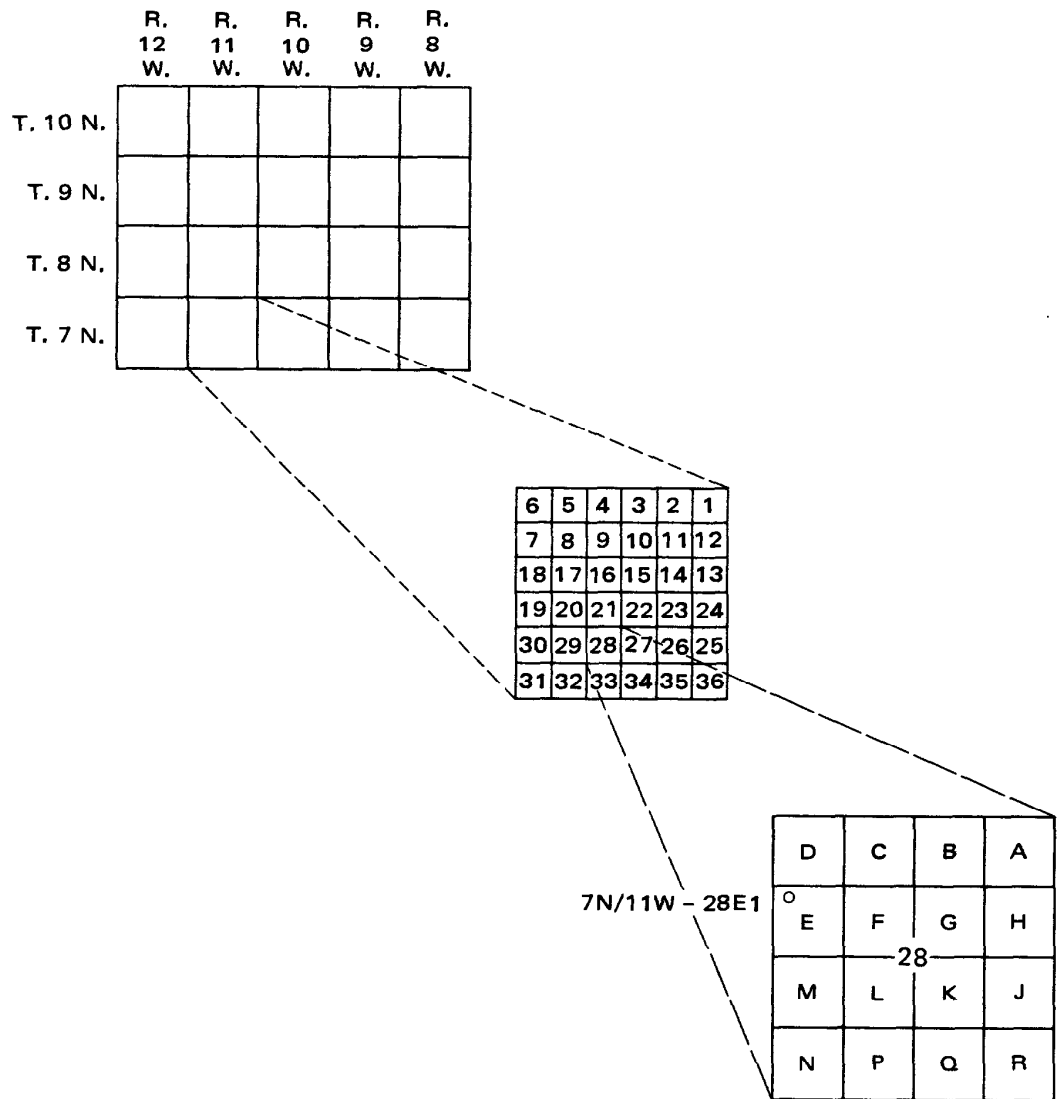
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## CONVERSION FACTORS

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[Factors for converting English units to metric units are shown to four significant figures. In the text, however, the metric equivalents are shown only to the number of significant figures consistent with the values for the English units]

<i>English</i>	<i>Multiply by—</i>	<i>Metric</i>
acres	$4.047 \times 10^{-1}$	ha (hectares)
acre-ft (acre-feet)	$1.233 \times 10^{-3}$	hm <sup>3</sup> (cubic hectometers)
acre-ft/yr (acre-feet per year)	$1.233 \times 10^{-3}$	hm <sup>3</sup> /yr (cubic hectometers per year)
(acre-ft/yr)/mi <sup>2</sup> (acre-feet per year per square mile)	$4.761 \times 10^{-4}$	(hm <sup>3</sup> /yr)/km <sup>2</sup> (cubic hectometers per year per square kilometer)
ft (feet)	$3.048 \times 10^{-1}$	m (meters)
ft/d (feet per day)	$3.048 \times 10^{-1}$	m/d (meters per day)
ft/mi (feet per mile)	$1.894 \times 10^{-1}$	m/km (meters per kilometer)
ft/yr (feet per year)	$3.048 \times 10^{-1}$	m/yr (meters per year)
ft <sup>2</sup> (square feet)	$9.290 \times 10^{-2}$	m <sup>2</sup> (square meters)
ft <sup>2</sup> /d (feet squared per day)	$9.290 \times 10^{-2}$	m <sup>2</sup> /d (meters squared per day)
in. (inches)	$2.540 \times 10^{-2}$	mm (millimeters)
in./yr (inches per year)	$2.540 \times 10^{-2}$	mm/yr (millimeters per year)
mi (miles)	1.609	km (kilometers)
mi <sup>2</sup> (square miles)	2.590	km <sup>2</sup> (square kilometers)



Well-numbering diagram.

# **CALIBRATION OF A MATHEMATICAL MODEL OF THE ANTELOPE VALLEY GROUND-WATER BASIN, CALIFORNIA**

By TIMOTHY J. DURBIN

## **ABSTRACT**

Antelope Valley is a closed topographic basin in the western part of the Mojave Desert in southern California. A ground-water basin with a surface area of 900 square miles (2,300 square kilometers) and a thickness of as much as 5,000 feet (1,500 meters) underlies the valley floor. The ground-water system consists of two alluvial aquifers separated by fine-grained lacustrine deposits. During the last 50 years, pumpage of ground water in excess of natural recharge has resulted in the steady decline of the ground-water level in the basin. The change in water level has been as much as 200 feet (61 meters). By 1972 the cumulative overdraft was about 9 million acre-feet (11,000 cubic hectometers). To help evaluate the possible impact of various water management alternatives, a mathematical model of the ground-water basin was constructed.

Construction of the ground-water model was the first part of a two-part study. The second part of the study will consist of the use of the model to evaluate the impact on the ground-water basin of various water-resource management alternatives. This report describes the mathematical model.

The model was calibrated by comparing the computed hydraulic heads to the corresponding prototype water levels for both steady-state and transient-state conditions. For the steady-state model the area-weighted median deviation of the computed hydraulic heads from the prototype water levels was 12 feet (3.7 meters). For the transient-state model the median deviation was 25 feet (7.6 meters).

The mathematical model is based on the governing equations of ground-water flow. The solution to the equations was approximated numerically by the Galerkin-finite element method.

## **INTRODUCTION**

Antelope Valley is a large topographic and ground-water basin in the western part of the Mojave Desert in southern California (pl. 1). Ground water has been the principal source of water for economic development in the valley. During the last 50 years, however, pumpage of ground water—chiefly for agricultural uses—in excess of natural recharge has resulted in the steady decline of the ground-water level in the basin. During this period, water levels in wells near



Lancaster have declined as much as 200 ft (61 m). By 1972 the cumulative overdraft was about 9 million acre-ft (11,000 hm<sup>3</sup>).

Antelope Valley is in the service area of the California Water Project. The Project comprises a major system of storage and conveyance facilities for exporting surplus water from northern California (and for transferring this water) to areas of deficiency elsewhere in the State (California Department of Water Resources, 1957). Because of the depletion of local ground-water supplies in Antelope Valley, the Antelope Valley-East Kern Water Agency, the Little Rock Irrigation District, and the Palmdale Water District have contracted for a combined maximum annual entitlement of 158,000 acre-ft (195 hm<sup>3</sup>) of imported water from the California Water Project. Deliveries of this water were begun in 1972, when 370 acre-ft (0.46 hm<sup>3</sup>) was supplied. Future deliveries will be increased gradually until the maximum entitlement is reached in about 1990 (California State Water Resources Control Board, 1974).

Various plans for the distribution and use of this water are being considered by the responsible water agencies. Plans are being considered also for the reclamation of waste water and the improved utilization of floodwater. To evaluate the possible impact of each alternative on the Antelope Valley ground-water basin, the U.S. Geological Survey and the California Department of Water Resources are engaged in a cooperative investigation.

The investigation was divided into two parts: (1) development of a mathematical model of ground-water flow and (2) use of the model to evaluate the impact of each water-management plan. This report describes the development of the mathematical model. The California Department of Water Resources plans to undertake the application of the model.

### WELL-NUMBERING SYSTEM

Wells are numbered according to their location in the rectangular system for subdivision of public land (see diagram, p. VI). For example, in the well number 7N/11W-28E1 the part (of the number) preceding the slash indicates the township (T. 7 N.); the number and letter following the slash indicate the range (R. 11 W.); the number following the hyphen indicates the section (sec. 28); the letter following the section number indicates the 40-acre (16-ha) subdivision of the section according to the lettered diagram (p. VI). The final digit is a serial number for wells in each 40-acre (16-ha) subdivision. The area covered by the report lies in the northwest quadrant of the San Bernardino base line and meridian and in the southeast quadrant of the Mount Diablo base line and meridian.

## DESCRIPTION OF THE STUDY AREA

### LOCATION AND GENERAL FEATURES

Antelope Valley lies in a westward-pointing wedge formed by the intersection of the San Andreas and Garlock fault zones (pl. 1). The valley is bordered on the northwest and north by the Tehachapi Mountains, the Rosamond Hills, and the Bissell Hills; on the southwest and south by the San Gabriel Mountains; and on the east by low hills and divides that separate the valley from upper Mojave Valley, Harper Valley, and Fremont Valley. Mountain and foothill land within Antelope Valley covers about 1,200 mi<sup>2</sup> (3,100 km<sup>2</sup>). Relatively flat valley land covers about 1,000 mi<sup>2</sup> (2,600 km<sup>2</sup>). The floor of the valley ranges from 2,300 to 3,500 ft (700 to 1,100 m) above sea level, thus lying at an altitude higher than most of the nearby desert valleys and considerably higher than the coastal plain to the south and the San Joaquin Valley to the north.

Antelope Valley is characterized by interior drainage that terminates at either Rosamond Lake or Rogers Lake playas. Broad alluvial fans extend as much as 15 mi (24 km) from the base of the mountains and hills that surround Antelope Valley.

The Antelope Valley ground-water basin covers about 900 mi<sup>2</sup> (2,300 km<sup>2</sup>). The basin is divided into ground-water subbasins by faults and other structural features. Subdivisions of the Antelope Valley ground-water basin are the Lancaster, Buttes, Pearland, Neenach, West Antelope, Finger Buttes, and North Muroc subbasins. The names and boundaries of the subbasins that were proposed by Bloyd (1967) are used in this report.

### GROUND-WATER GEOLOGY

The Antelope Valley ground-water basin occupies part of a structural depression that has been downfaulted between the Garlock and San Andreas fault zones. The effect of the faulting was to stimulate erosion of the hills and mountains that surround the valley. The area presently occupied by the ground-water basin became the receptacle for the eroded materials. Economically important aquifers within the ground-water basin occur in the sedimentary deposits that were formed by the deposition of the eroded materials. These deposits have accumulated to a thickness locally of as much as 8,000 ft (2,400 m) (Mabey, 1960).

Consolidated, virtually non-water-bearing rocks crop out in the highlands that surround the ground-water basin (pl. 1). These rocks also underlie and form the bottom of the ground-water basin. The consolidated rocks consist of igneous and metamorphic rocks, which

form the basement complex of the study area, and of indurated continental rocks that are interbedded with volcanic flows. The basement complex is of pre-Tertiary age, and the continental rocks are of Tertiary age (Dibblee, 1967).

Water-bearing, mostly unconsolidated deposits that contain sufficient water for economic use overlie the consolidated rocks. The unconsolidated deposits consist of alluvium of Pliocene to Holocene age and of lacustrine deposits of Pleistocene to Holocene age (Dutcher and Worts, 1963) which are interbedded with the alluvium.

#### *Alluvium.*

The alluvium is composed of unconsolidated to moderately indurated, poorly sorted gravel, sand, silt, and clay. Older units of the alluvium are more compacted and indurated, somewhat coarser grained, more weathered, and more poorly sorted than the younger units. The hydraulic conductivity of the alluvium decreases with increasing age (Dutcher and Worts, 1963) and, consequently, with increasing depth.

Dutcher and Worts (1963) identified seven lithographic units within the alluvium. These units are older fan deposits, older alluvium, younger fan deposits, younger alluvium, lakeshore deposits, old wind-blown sand, and dune sand. The older fan deposits comprise old moderately to highly indurated conglomerate and stream-channel deposits that yield little water to wells. The older alluvium comprises the coarse-grained, weathered, and moderately well-sorted alluvium that underlies the valley areas beneath the younger alluvium. The older alluvium is locally as much as 5,000 ft (1,500 m) thick, and these deposits constitute the bulk of the water-bearing deposits in the Antelope Valley ground-water basin. The younger fan deposits commonly are composed of very poorly sorted boulders, gravel, sand, silt, and clay. The younger alluvium is composed predominantly of sand and gravel. Prior to about 1945, the younger alluvium was the main source of ground water for agriculture in the Lancaster subbasin, but since that time it has been substantially dewatered. The lakeshore deposits, the old wind-blown sand, and the dune sand are above the regional water table and do not contain significant quantities of ground water.

#### *Lacustrine deposits.*

During the depositional history of the Antelope Valley ground-water basin, a large lake occupied parts of the Lancaster and North Muroc subbasins. Fine-grained lacustrine deposits formed in this lake.

The depositional environment of the lacustrine deposits has varied (Dutcher and Worts, 1963). During pluvial periods, or times of rela-

tively heavy precipitation, massive beds of blue clay formed in deep, perennial lakes. At least two pluvial periods have been followed by interpluvial periods, during which playa and similar deposits formed in shallow, intermittent lakes. Individual clay beds are locally as much as 100 ft (30 m) thick. These are interbedded with lenses of coarser material as much as 20 ft (6.1 m) thick. The clay yields virtually no water to wells, but interbedded materials supply some water to wells.

During deposition of the lacustrine deposits, alluvial debris that was supplied from the San Gabriel Mountains encroached upon the lake, forcing it northward and causing the northward transgression of alluvium over lacustrine deposits. Near the southern limit of the Lancaster subbasin, the lacustrine deposits are buried beneath as much as 800 ft (240 m) of alluvium, but near the northern limit the lacustrine deposits are exposed at the land surface (pl. 1).

The subsurface extent of the lacustrine deposits is shown on plate 1. These deposits underlie the central part of the Lancaster subbasin and the southwestern part of the North Muroc subbasin. They extend from near Little Buttes on the west to the east edge of Rogers Lake and from near the southern limit of the Lancaster subbasin on the south to the north edge of Rogers Lake.

The buried body of lacustrine deposits has a somewhat lenticular shape. The thickest section occurs near the center of the Lancaster subbasin (pl. 1), and the unit thins toward its edges. Near Little Buttes and near the east and north edges of Rogers Lake, the unit thins to extinction. Along the northern and southern boundaries of the Lancaster subbasin, the lacustrine deposits terminate against buried escarpments that have formed on the consolidated rocks; the thicknesses along these boundaries are 100 ft (30 m) and 250 ft (76 m), respectively.

#### *The principal and deep aquifers.*

Two major aquifers occur within the Antelope Valley ground-water basin: the principal and the deep aquifers (Dutcher and Worts, 1963). The lacustrine deposits separate these aquifers both vertically and horizontally.

The principal aquifer, which supplies nearly all water pumped from wells in the Antelope Valley ground-water basin, overlies the lacustrine deposits (pl. 1) and is unconfined. This aquifer extends over the area to the south and west of Rogers Lake and includes the Neenach, West Antelope, Finger Buttes, Buttes, and Pearland subbasins and part of the Lancaster subbasin (pl. 1).

The deep aquifer, in part, underlies the lacustrine deposits. The extent of this aquifer includes the area of the lacustrine deposits and the area east and north of Rogers Lake. This area includes the North

Muroc subbasin and part of the Lancaster subbasin (pl. 1). In the area where the deep aquifer is overlain by the lacustrine deposits, the aquifer is confined; in other areas it is unconfined.

#### GROUND-WATER MOVEMENT

Ground water in the Antelope Valley ground-water basin moves centripetally from the base of the San Gabriel and Tehachapi Mountains toward the north-central part of the Lancaster subbasin (pl. 2). Before the extensive pumping of ground water, the water table for the principal aquifer was near land surface in the north-central part of the Lancaster subbasin, and ground-water discharge occurred because of direct evapotranspiration of ground water in this area. Pumping of ground water and the subsequent increase in depth to the water table stopped this discharge.

Ground water in the Neenach, West Antelope, and Finger Buttes subbasins moves into the Lancaster subbasin. At the western limit of the lacustrine deposits, part of this water moves over the lacustrine deposits and within the principal aquifer, and part moves under the lacustrine deposits and within the deep aquifer.

Ground water in the Buttes and Pearland subbasins also moves into the Lancaster subbasin. The upper surface of the lacustrine deposits is below the path of the inflowing water, however, and this water moves into the Lancaster subbasin wholly over the top of the lacustrine deposits and within the principal aquifer.

In the Lancaster subbasin, subsurface discharge of ground water in the principal aquifer is impeded by consolidated rocks on the east and north and by the lacustrine deposits on the northeast. Before the 1940's, ground water in the deep aquifer moved northward out of the Lancaster subbasin, under the lacustrine deposits, and into the North Muroc subbasin. By 1961, the direction of ground-water movement in the deep aquifer had been reversed in the area underlying and immediately south of Rogers Lake, and the direction of ground-water movement there is now southward toward the center of the Lancaster subbasin (pl. 3). North of Rogers Lake, ground water moves from the north Muroc subbasin into Fremont Valley.

Reversal of the direction of ground-water movement in the area south of Rogers Lake was caused for the most part by pumping ground water from the principal aquifer. This pumping also produced significant changes from 1915 to 1961 in water levels in the principal aquifer (pls. 2, 3), especially in the Lancaster subbasin. The main change was the development of areas of low water levels near the west and east sides of the Lancaster subbasin.

Leakage of ground water between the principal and deep aquifers occurs through the lacustrine deposits. Based on hydraulic heads for the principal and deep aquifers that were computed by the mathemat-

ical model for both steady-state and transient-state conditions, the direction of leakage is downward from the principal aquifer into the deep aquifer along the western and southern periphery of the lacustrine deposits. In the north-central part of the area underlain by lacustrine deposits, the direction of leakage historically was upward from the deep aquifer into the principal aquifer. Because of pumping of ground water from the principal aquifer, the area in which upward leakage occurs is now more toward the south in the areas of concentrated pumping.

Major faults in the Antelope Valley, especially the Randsburg-Mojave fault, act as partial barriers to the movement of ground water. Water-level differentials of as much as 300 ft (91 m) occur across the Randsburg-Mojave fault. Along several other faults that cross the Antelope Valley ground-water basin the water table is several tens of feet higher on the upgradient side of the fault than on the downgradient side. The studies of faults near Long Beach, Calif., by Poland, Piper, and others (1956) and near San Bernardino, Calif., by Dutcher and Garrett (1963) indicate that some possible causes of the barrier effect along faults cutting alluvial deposits are (1) local and incomplete offsetting of sand beds against clay beds; (2) sharp local folding of beds near the faults, causing relatively impermeable clay beds to be turned across the direction of ground-water movement; (3) cementation of gravel and sand beds immediately adjacent to the fault by deposition of carbonate minerals from water moving along the fault plane; and (4) development of secondary clayey gouge zones along the faults.

### THE MATHEMATICAL MODEL

A conceptual approach to ground-water modeling was used in this study. First, a conceptual model of the ground-water system, which represents the reduction of the prototype to its essential elements, was developed. Then a mathematical analog, or mathematical model, of the conceptual model was constructed. The mathematical model is a good approximation of the physical processes that were assumed to operate in the conceptual model, but it is only an approximate representation of the prototype.

The conceptualization of the prototype must be simplified to the extent that an operational mathematical model can be constructed; however, simplification must not be so great that the essential characteristics of the prototype are not retained. In practice, our ability to construct mathematical models is limited, and this situation requires that we correspondingly adjust our expectations of the model. We would like a model that represented all characteristics of the prototype but must settle for a model that represents a few of its more important characteristics.

The mathematical model of the Antelope Valley ground-water

basin is described in following sections. When the model is being discussed, the question of scale invariably arises. It is therefore important to emphasize that this study is being carried out on a megascopic scale. Physical properties and processes observable on a scale of several miles or greater are being considered.

The mathematical model developed for the Antelope Valley ground-water basin treats the prototype as a two-aquifer system. The aquifers are linked together in the model through a leakage term that represents the flow through the lacustrine deposits. As mentioned earlier, the modeling of a ground-water system is accomplished by substituting a simplified conceptual model for the prototype. Some of the more important simplifying assumptions that relate directly to the mathematical model are:

1. Ground-water movement within an aquifer is strictly horizontal.
2. Ground-water movement within the lacustrine deposits is strictly vertical.
3. Hydraulic head changes within the lacustrine deposits do not cause corresponding changes in the volume of water that is stored in these deposits.
4. Changes in ground-water storage in the aquifers occur instantaneously with changes in hydraulic head.
5. The physical parameters of the system do not change with the state of the system.
6. The aquifers are bounded by an impermeable boundary.
7. Recharge occurs instantaneously.
8. The aquifers are isotropic.
9. The barrier effect of faults can be represented by a zone of low transmissivity.

The general equation that approximately governs the flow of water in a two-dimensional isotropic aquifer is

$$\frac{\partial}{\partial x} \left( T \frac{\partial h}{\partial x} \right) - \frac{\partial}{\partial y} \left( T \frac{\partial h}{\partial y} \right) - S \frac{\partial h}{\partial t} - W - \frac{K}{b}(h - h_a) = 0, \quad (1)$$

where  $T$  is the transmissivity of the aquifer,  $h$  is the hydraulic head in the aquifer,  $S$  is the storage coefficient of the aquifer,  $W$  is the flux of a source or sink,  $K$  and  $b$  are the vertical hydraulic conductivity and the thickness of the lacustrine deposits, and  $h_a$  is the hydraulic head in the adjacent aquifer.

The governing equation was solved on triangular elements by the Galerkin-finite-element method. Briefly, the method involved dividing the aquifers into elements having triangular shapes (pls. 4, 5 show the element configurations used for the principal and deep

aquifers) and assuming that the solution to the governing equation can be expressed as a linear combination of relatively simple trial functions. Associated with the trial functions are coefficients that the Galerkin computational scheme adjusts in order to give some best approximation to equation 1. The Galerkin-finite-element scheme is described more completely in the section "Numerical Solution of the Ground-Water Equations." A computer program that embodies this solution scheme was written especially for this study.

The geometrical relations in the ground-water basin are specified in the model through the configuration of elements. The physical properties of the prototype are specified in the model by assigning parameter values to the elements. These values represent the prototype transmissivity, storage coefficient (for the transient-state model), and, where appropriate, the thickness and vertical hydraulic conductivity of the confining member. The model uses the above specifications to compute hydraulic heads that mathematically satisfy the physical parameters of the system and also satisfy the rate of inflow and outflow that is applied.

One important source of uncertainty in the model is the unavoidable lack of definitive measurements of the model parameters. The aggregate character of these parameters makes laboratory measurements of little use. Current methods of field testing, such as aquifer tests, are of limited use in providing values that can be used directly or extrapolated reliably to the large-scale phenomena simulated by the model.

To improve the prior estimates of these parameters, the model was calibrated by iteratively adjusting the parameter values until the model reproduced historical conditions to an acceptable degree. The model was calibrated to two different historical conditions, first to a steady-state condition and second to a transient-state condition. These calibrations were subjective and, to a large extent, based on trial and error.

### STEADY-STATE MODEL

Prior to the entry of man into Antelope Valley, the ground-water basin was in an equilibrium or steady-state condition: recharge equaled discharge and, considering periods of several years, the water levels in the ground-water basin remained unchanged with time. Several hundred wells were drilled in Antelope Valley prior to 1908, but the wells were used primarily to secure patents to government land (Snyder, 1955). The significant use of ground water for irrigation began in about 1915, and before this date the ground-water basin can be considered to have been in an equilibrium state.

The steady-state model of the Antelope Valley ground-water basin is intended to represent this condition. Input to the steady-state



model is the natural recharge and discharge of ground water. Output from the model is the primordial hydraulic heads in the principal and deep aquifers. The calibration problem for this model was to refine prior estimates of the transmissivity of the principal and deep aquifers and prior estimates of the vertical hydraulic conductivity of the lacustrine deposits that separate these aquifers.

#### NATURAL RECHARGE

##### *Occurrence of natural recharge.*

The Antelope Valley ground-water basin is recharged naturally by infiltration of streamflow that originates in the mountain areas contiguous to the ground-water basin. For the most part, streamflow that enters the valley is ephemeral. During storm periods, streamflow debouches along the valley perimeter and moves down the alluvial fans and toward Rosamond and Rogers Lake playas. As streamflow moves down the alluvial fans, it infiltrates the permeable surficial deposits on the fans and seldom reaches the playas. The infiltrate is partly evaporated and partly transpired by riparian vegetation. The remainder percolates through the alluvial deposits until it reaches the water table.

Because the average annual precipitation on the valley floor is less than 10 in. (250 mm) (Rantz, 1969), very little runoff is generated on the valley floor, and probably very little precipitation penetrates below the root zone. In an environment somewhat similar to that of Antelope Valley, Blaney, Taylor, and Young (1930) and Young and Blaney (1942) found that precipitation does not penetrate below the root zone if the annual precipitation is less than about 12 in. (300 mm). Therefore, precipitation on the valley floor was not considered to be an important source of ground-water recharge.

In the mountain areas the average annual precipitation is generally greater than 12 in. (300 mm) (Rantz, 1969). Part of this precipitation becomes surface runoff, and part becomes soil moisture. For most of the mountain areas, precipitation that infiltrates the soil mantle is in excess of the moisture requirements of vegetation and soil evaporation. Much of the surplus soil moisture moves along the subsurface contact between a thin soil mantle and the underlying bedrock. This water moves downslope and eventually may reach the ground-water basin.

Two important sources of ground-water recharge are possible: streamflow infiltration and near-surface horizontal percolation. The net recharge of ground water from both sources is equal to the total surface-water flow onto the valley floor, plus the total subsurface inflow, minus the total quantity of water removed from stream channels by evapotranspiration. Unfortunately, practical techniques are not available for estimating the subsurface inflow or the quantity of

evapotranspiration from the stream channels. The assumption that was made is that these two quantities are locally equal and that the local net recharge is numerically equal to the surface-water discharge from the mountains onto the valley floor.

*Mean annual streamflow.*

The drainage area tributary to the Antelope Valley ground-water basin is about 385 mi<sup>2</sup> (1,000 km<sup>2</sup>). Runoff from about 20 percent of this area is gaged. Runoff records (table 1) are available for Big Rock Creek near Valyermo, Little Rock Creek near Little Rock, and Santiago Creek above Little Rock Creek, all in the San Gabriel Mountains (pl. 6). The collective mean annual discharge at these points is about 24,300 acre-ft (30.0 hm<sup>3</sup>). The mean annual runoff from other areas of Antelope Valley was estimated by using a method that is based on the measurement of the width and average depth of stream channels at bars and berms.

An alluvial channel adjusts in size to accommodate the discharge it receives (Moore, 1968; Leopold and Wolman, 1957). Although the channel geometry is influenced by the channel slope and pattern, sediment loads, cohesiveness of the banks, and vegetation, studies by Moore (1968) indicate that the dimensions of cross sections at the bars and berms are not significantly affected by these factors and that the dimensions of cross sections are related to the mean annual runoff. Using the width ( $W$ ) and depth ( $D$ ) in feet at bars and berms, Hedman (1970) developed, from southern California streamflow data, the empirical relation

$$Q = 258 W^{0.80} D^{0.60} \quad (2)$$

for estimating the mean annual discharge ( $Q$ ) in acre-feet. The standard error of estimate for the relation was 29 percent.

The channel-geometry relation was used to estimate the mean annual discharge for 25 ungaged streams. Channel geometry was measured in 11 stream channels in the San Gabriel Mountains and 14 stream channels in the Tehachapi Mountains. The cumulative drainage area above the measurement points is about 27 percent of the ungaged tributary area in the San Gabriel Mountains and about 40 percent of the drainage area in the Tehachapi Mountains.

The estimated discharge at the channel-geometry measurement locations was extrapolated to other ungaged areas by the relation

$$Q = CA, \quad (3)$$

where  $Q$  is the mean annual runoff,  $A$  is the drainage area, and  $C$  is the average ratio of runoff to the drainage area at the channel-geometry-measurement locations. The value of  $C$  for the San Gabriel Mountains (fig. 1) was 50 (acre-ft/yr)/mi<sup>2</sup> [0.024 (hm<sup>3</sup>/yr)/km<sup>2</sup>]. The value of  $C$  for the Tehachapi Mountains (fig. 2) was 60 (acre-ft/yr)/mi<sup>2</sup> [0.029 (hm<sup>3</sup>/yr)/km<sup>2</sup>].

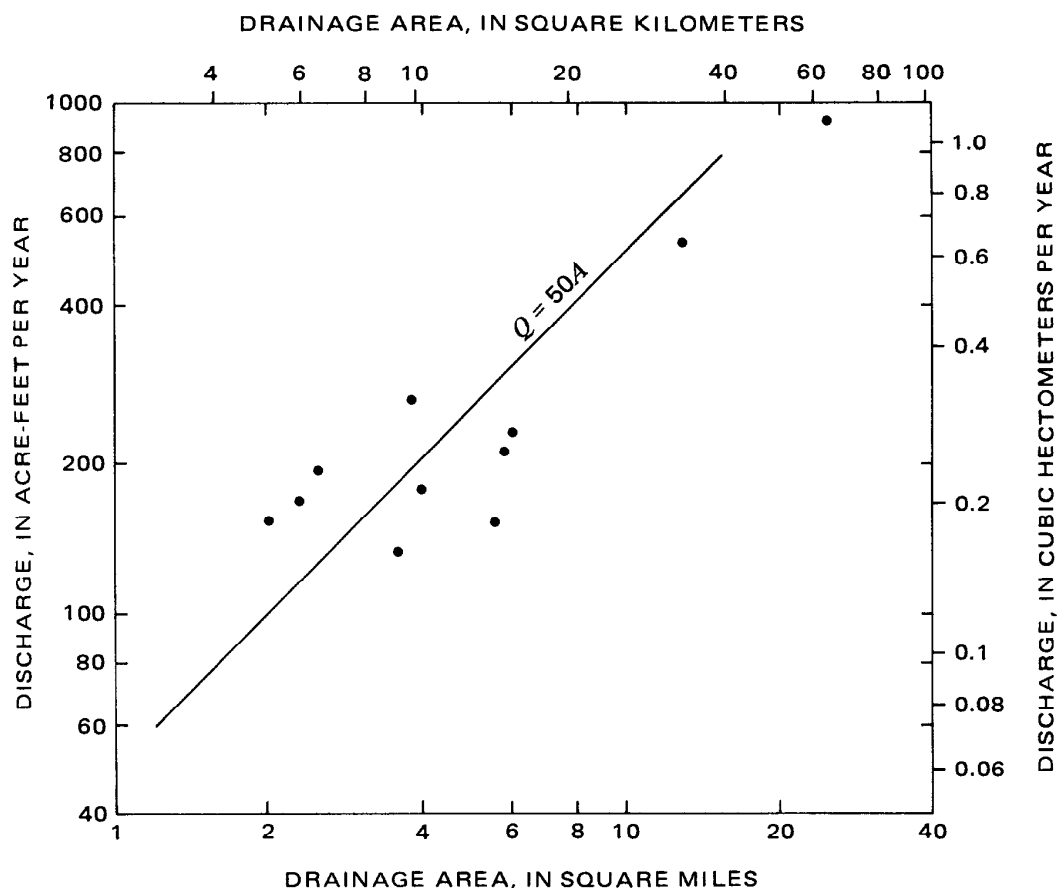


FIGURE 1.—Relation of stream discharge to drainage area for the San Gabriel Mountains.

By using the coefficient values, the cumulative average annual runoff from the ungaged drainage basins was estimated to be 16,400 acre-ft (20.2 hm<sup>3</sup>) (table 1). The total average annual runoff from

TABLE 1.—Average annual runoff to Antelope Valley

Drainage basin	Area (mi <sup>2</sup> )	Runoff (acre-ft/yr)
Measured discharge:		
Big Rock Creek .....	23	11,500
Little Rock Creek .....	49	12,100
Santiago Creek .....	11	700
Estimated discharge:		
San Gabriel Mountains .....	174	8,700
Tehachapi Mountains .....	128	7,700
Total runoff .....		40,700

both gaged and ungaged drainage basins is 40,700 acre-ft (50 hm<sup>3</sup>). Natural recharge to the ground-water basin, which was assumed to be numerically equivalent to runoff, was distributed geographically as shown on plate 6.

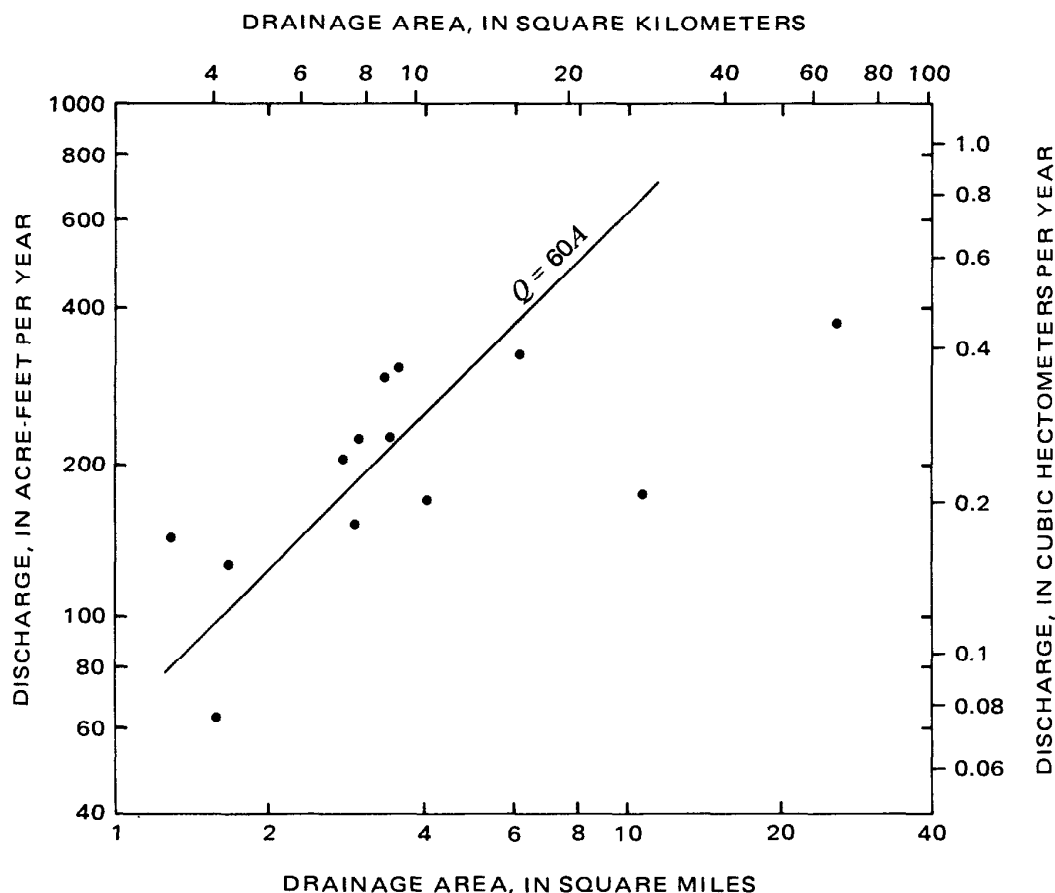


FIGURE 2.—Relation of stream discharge to drainage area for the Tehachapi Mountains.

#### NATURAL DISCHARGE

Average annual discharge from ground water over an extended period of time will equal average annual recharge when there is no interference by man. Because of heavy pumping, however, natural discharge has been substantially reduced. Prior to the pumping of ground water, natural discharge occurred by subsurface outflow, by evapotranspiration, and by springs. Subsurface outflow and evapotranspiration were the principal mechanisms for natural discharge. The discharge of springs was not significant and was probably less than 300 acre-ft/yr (0.37 hm³/yr) (Thompson, 1929; Johnson, 1911).

##### *Subsurface outflow.*

North of Rogers Lake, the land surface along the divide between Antelope Valley and Fremont Valley is less than 100 ft (30 m) higher than the lowest point in Antelope Valley. Although consolidated rocks crop out on both sides, the divide for a width of about 1 mi (1.6 km) is underlain by as much as 1,000 ft (300 m) of unconsolidated deposits. At this location some ground water is discharged from

the Antelope Valley ground-water basin into the Fremont Valley ground-water basin as subsurface outflow (pl. 6).

The quantity of subsurface outflow can be approximated by the relation

$$Q = AK \left( \frac{\partial h}{\partial n} \right), \quad (4)$$

where  $Q$  is the subsurface outflow,  $A$  is the cross-sectional area of flow,  $K$  is the hydraulic conductivity of the unconsolidated deposits, and  $\partial h / \partial n$  is the hydraulic-head gradient. Based on the subsurface projection of the exposed consolidated rocks beneath the unconsolidated deposits and on the measurement of the depth to ground water, the cross-sectional area of flow is about  $1.2 \times 10^6$  ft<sup>2</sup> ( $1.1 \times 10^5$  m<sup>2</sup>). Aquifer test data (Moyle, 1965) indicate that the hydraulic conductivity of the unconsolidated deposits is about 50 ft/d (15 m/d). The water-level gradient is about 10 ft/mi (1.9 m/km). The substitution of these values into equation 4 gives an estimated subsurface outflow of 1,000 acre-ft/yr (1.2 hm<sup>3</sup>/yr) (pl. 6).

#### *Evapotranspiration.*

Large areas of alkali soil in the Lancaster subbasin (pl. 6) indicate a former discharge of ground water by evapotranspiration (Carpenter and Cosby, 1926). The alkali was dissolved in ground water, and as the result of evapotranspiration the alkali and other dissolved solids were precipitated out of solution at or near the land surface.

Ground-water discharge by evapotranspiration generally occurs when the water table is within about 10 ft (3 m) of the land surface. Under this condition some plant species obtain their water supply from either the ground water or the capillary fringe, and the consumption of ground water by this vegetation is an important mechanism for ground-water discharge. If the water table is within a foot or so of land surface, significant quantities of ground water may additionally be discharged by direct evaporation of water from the capillary fringe. The mass balance for the Antelope Valley ground-water basin indicates that the annual discharge of ground water by evapotranspiration may have been about 39,400 acre-ft (48.5 hm<sup>3</sup>).

Where a linear relation between the depth to the water table and the ground-water discharge is assumed, the relation can be defined if two points on the relation are specified. For example, salt grass (*Distichlis stricta*) was the principal plant species in the area of evapotranspiration in Antelope Valley (Thompson, 1929). Robinson (1958) reported that for a depth to the water table of 1 ft (0.3 m) evapotranspiration from salt grass may be as much as 75 percent of the pan evaporation. The pan evaporation in Antelope Valley is about 114

in./yr (2,900 mm/yr) (Bloyd, 1967), and 75 percent of this value is 86 in./yr (2,180 mm/yr). Lysimeter studies by Lee (1912) indicate that evapotranspiration from salt grass virtually stops if the depth to the water table is greater than 10 ft (3.0 m). The data from Robinson and Lee give two points on the depth-discharge relation for salt grass, and, given the assumption above, these data are sufficient to define the linear relation that is shown in figure 3.

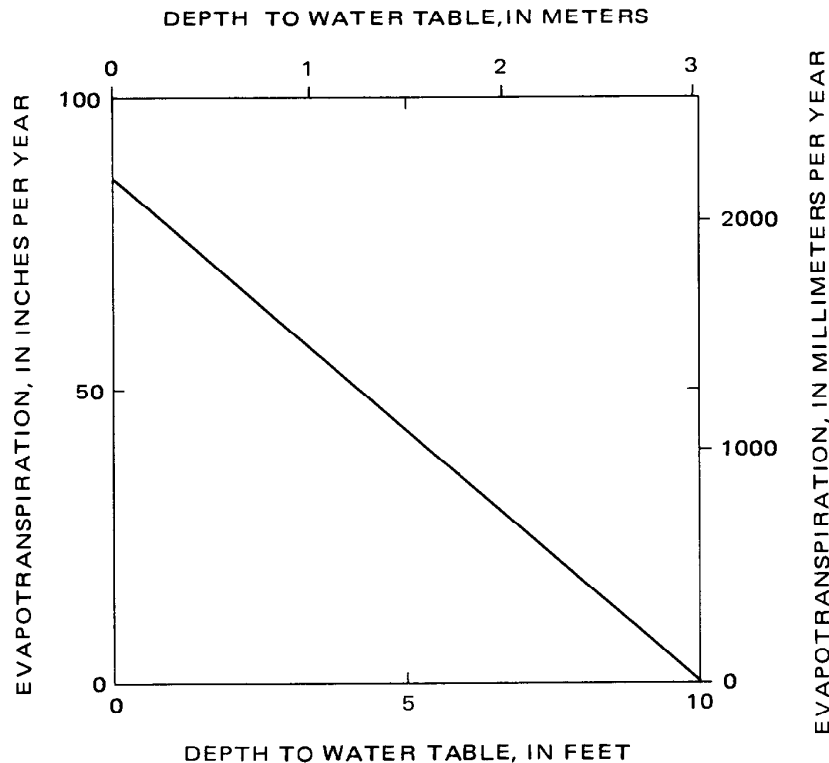


FIGURE 3.—Relation of evapotranspiration to depth to water table.

#### CALIBRATION OF THE STEADY-STATE MODEL

The steady-state model was calibrated to the estimated prototype water levels for 1915 (pl. 2). Most of the first wells in Antelope Valley were drilled in the Lancaster subbasin. Consequently, most of the early water-level measurements were made in wells that were located there. For these wells, Johnson (1911) reported water-level measurements that he made in the winter of 1908–09. Thompson (1929) reported water-level measurements that he and others made during 1907–21. In most instances, these water-level measurements can reasonably be assumed to represent conditions existing in 1915.

Few early water-level measurements are available for the area outside the Lancaster subbasin, but water levels in much of this area have not changed more than a few tens of feet since 1915. Water-level measurements that were made as late as 1965 (Dutcher and others,

1962; Moyle, 1965; Koehler, 1966) were assumed to represent the water-level conditions existing in 1915. Nevertheless, in the area outside the Lancaster subbasin, the geographic distribution of available water-level measurements is not complete, and the estimated water levels in this area were based mainly on the subjective extrapolation of sparse data.

The measured water levels represent for the most part the water-level conditions in the principal aquifer. In the part of Antelope Valley south and west of Rogers Lake, no field data are available that indicate the hydraulic head in the deep aquifer. Some water-level measurements are available for wells in the deep aquifer in the area north and east of Rogers Lake.

In addition to requiring estimates of the water levels, the calibration procedure requires that initial estimates be made of the transmissivity of the principal and deep aquifers and of the vertical hydraulic conductivity of the lacustrine deposits.

The initial estimates of the transmissivity of the principal aquifer were based on specific-capacity data reported by Bloyd (1967). Transmissivity of the aquifer can be estimated by multiplying the specific capacity of a properly constructed well by a factor (Theis, 1963). If homogeneous units of measure are used for both the specific capacity and the transmissivity, the factor is dimensionless, and its value ranges between 1.0 and 1.4. The correct value of the factor depends in part on the duration of the pumping tests used to estimate the specific capacity of a well. The data reported by Bloyd (1967) are based on pumping tests of short duration, and a value of 1.0 was used for the factor.

Field data are not available for estimating the transmissivity of the deep aquifer, except in the vicinity of Rogers Lake where some wells penetrate this aquifer. In other areas of the valley, data on transmissivity are not available from the wells that penetrate the deep aquifer. The specific-capacity data reported by Bloyd (1967) were used to estimate transmissivity in the limited area for which these data are available. Transmissivity for the deep aquifer in other areas was prescribed subjectively.

The vertical hydraulic conductivity of the lacustrine deposits was estimated from sparse field data. Based on the probable properties of the lacustrine deposits, a value of  $10^{-2}$  ft/d ( $3 \times 10^{-3}$  m/d) was assumed for the vertical hydraulic conductivity of the lacustrine deposits. Using this value in the model, computed head differences between the aquifers were comparable to those presumed to have existed in the prototype, which are generally less than 20 ft (6.1 m), and the value was not changed during calibration.

Hydraulic heads in the principal and deep aquifers were computed using estimates of the system parameters that are described above.

Originally these heads deviated locally as much as 500 ft (150 m) from the prototype water levels. The objective of the calibration was to reduce the local deviations to a reasonable level by adjusting the system parameters within a range of physically plausible values.

During calibration of the steady-state model, adjustments were made primarily to the transmissivity of the principal aquifer. Twenty-two calibration runs were made. During the early runs, gross adjustments were made to the transmissivity of large areas. Finer adjustments were made to the transmissivity over smaller areas during the later calibration runs. The net effect of these adjustments was to increase the transmissivity by about 15 percent above the initial estimates (fig. 4). The adjusted transmissivity of the principal aquifer is shown on plate 7, and the adjusted transmissivity of the deep aquifer is shown on plate 8.

Plate 9 shows hydraulic heads in the principal and deep aquifers computed by the mathematical model using the transmissivity distributions shown on plates 7 and 8. The shape of the computed solu-

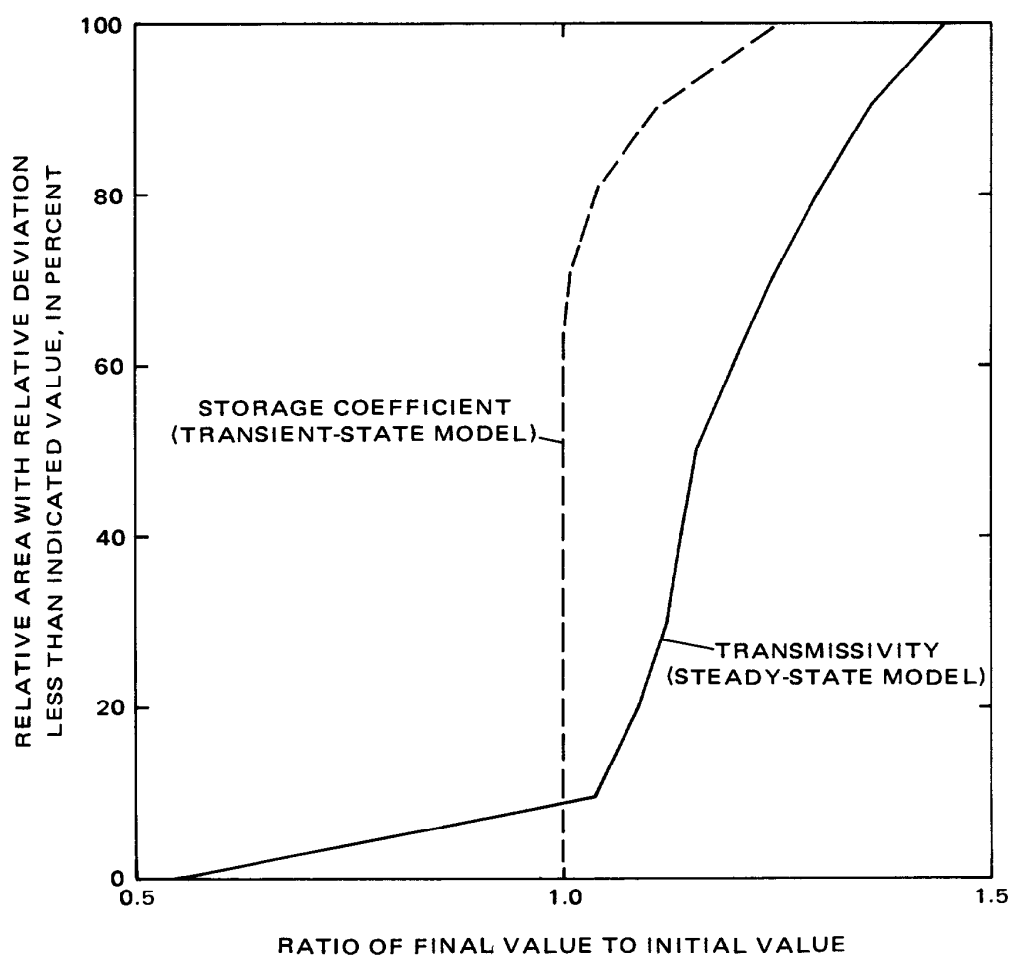


FIGURE 4.—Relative cumulative distribution of the relative deviation of the model parameters from their initial values.



tion compares well with the potentiometric map of the prototype water levels for the principal aquifer shown on plate 2. The area-weighted median absolute deviation of computed hydraulic heads from prototype water levels is 12 ft (3.7 m) (fig. 5). The largest devia-

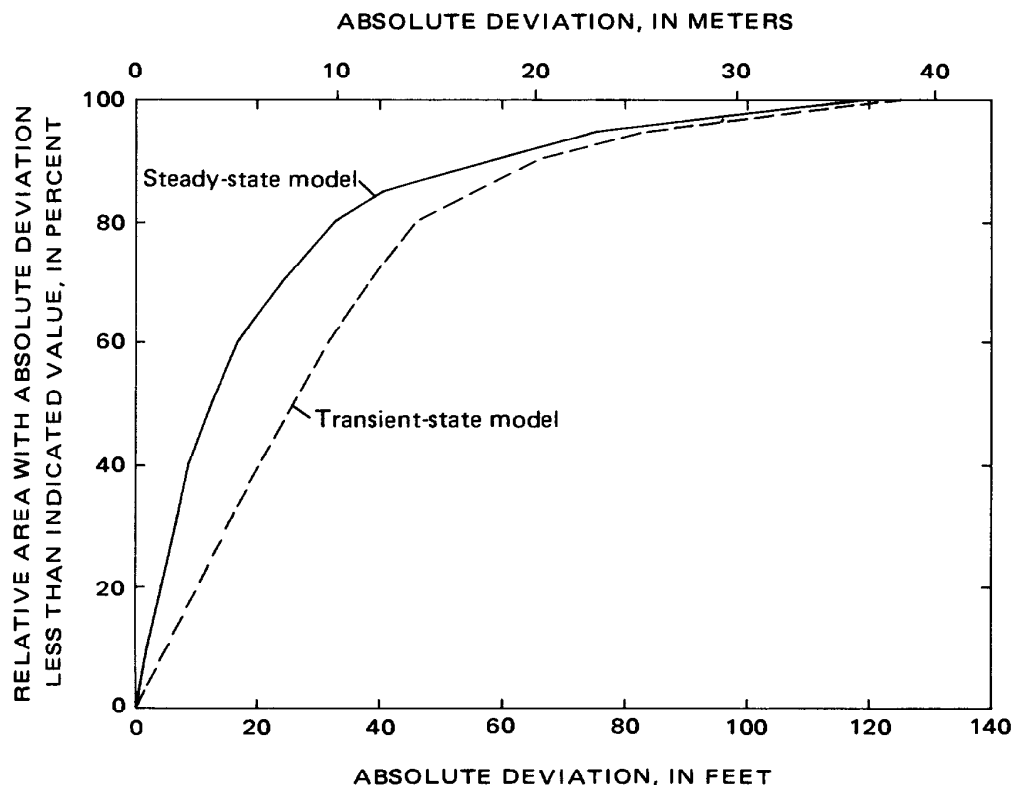


FIGURE 5.—Relative cumulative distribution of the absolute deviation of the computed hydraulic head from the prototype water level for the principal aquifer.

tions occur in areas where sparse field data introduce considerable uncertainty into the estimates of the prototype water levels. Field data are available for most of the Lancaster subbasin, and for this area the median absolute deviation was 7 ft (2.1 m) (fig. 6).

### TRANSIENT-STATE MODEL

The use of ground water in Antelope Valley for agriculture disturbed the primordial equilibrium in the ground-water basin. Over much of the period of ground-water use, the net extraction of ground water has been in excess of the net natural recharge of ground water. As a result, the overall ground-water trend in Antelope Valley has been one of declining water levels. Hydrographs of wells perforated in the principal aquifer indicate that from 1920 through 1972 the water level in this aquifer declined as much as 200 ft (61 m). The rate of decline has been as much as 4 ft/yr (1.2 m/yr).

The transient-state model of the Antelope Valley ground-water